Inter-Nuclear $PE$ Graphs: Beyond Balancing Reaction Equations and Calculating Mass Defects in Analyzing Fusion, Fission, and $\alpha$ Decay

**Outline**

- Motivation for internuclear $PE$ graphs
  - Connection with interatomic $PE$ graphs
  - Characterizing certain nuclear processes

- Construction of internuclear $PE$ graphs
  - General features
  - Exothermic fission example
  - Exothermic fusion example

- Analysis of nuclear processes
  - Fusion/fission initiation energy paradox
  - $\alpha$ decay versus "symmetric fission" paradox

- Conclusions
  - So what can internuclear $PE$ graphs do?

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**Motivation for internuclear $PE$ graphs**

Connection with interatomic $PE$ graphs

Interatomic $PE$ graphs
- A.k.a. "Lennard-Jones" plots
- Characterizes atom-atom interactions
- $PE$ gradient (slope) = force

- Similarly, an internuclear $PE$ graph can be constructed to characterize nuclear-nuclear interactions

Emphasis on describing "what happens," not explaining "why?"

Two "toy" models can motivate the "why" of nuclear processes!

Fusion
Fission
$\alpha$ decay
$\beta^+$ ($\epsilon$) decay
$\gamma$ decay

"Different model approaches try to accentuate various aspects of nuclear structure in a simple and schematic way. No single model, as yet, is detailed enough to encompass all aspects of the nucleus..."

— K. Heyde,
*Basic Ideas and Concepts in Nuclear Physics*, 1994
• Construction of internuclear PE graphs
  General features

  Internuclear PE graphs
  • Characterizes nuclear-nuclear interactions
  • PE gradient (slope) = force
  Electric repulsion when separated
  Combination of strong nuclear attraction and electric repulsion when touching (net force attractive)

  Exothermic fusion example

  Exothermic fusion reaction:
  \( ^3\text{He} + ^2\text{H} \rightarrow ^4\text{He} + n + [2.81 \times 10^{12} \text{ J}] \).

  Construction of internuclear PE graph:
  • Radii calculated from \( R = (1.2 \text{ fm}) A^{1/3} \).
  • Energy at scission point calculated from \( PE_{\text{sc}} \) of daughter nuclei
  • Assume straight-line segment during elongation

  What can this graph tell us?
  • PE gradient (slope) = net force between daughter nuclei during elongation or separation
  • Required initiation energy from neutron (overestimated, if tunneling effects are not included, and graph not smoothed at scission point)

• Analysis of nuclear processes
  Fusion/fission initiation energy paradox

  What can these graphs tell us?
  • Both graphs constructed in the same manner
  • Appreciation of relative distance and energy scales involved

  Common question asked by students:
  • Initiation energy for tritium-deuterium fusion is two orders of magnitude smaller than initiation energy for induced U-238 fission!
  • Yet why is fusion so much more difficult to initiate than fission?

  Fusion—two nuclei must come into contact
  • Both nuclei repel each other
  • Both nuclei move independent of each other in plasma phase

  Induced fission—neutron penetrates U-238 nucleus
  • No neutron-uranium repulsion
  • Neutron will eventually hit a U-238 nucleus in a solid phase critical mass sample
• Analysis of nuclear processes
  \( \alpha \) decay versus "symmetric fission" paradox

Common question asked by students:
• \( \alpha \) decay can be thought of as "asymmetric" fission, where less net energy released than fission into symmetric daughter nuclei.
• Yet why is \( \alpha \) decay observed rather than energetically favorable symmetric fission?

Compare internuclear **PE** graphs of both processes:
• Symmetric fission: more exothermic, but higher initiation energy (\( KE \) from external neutron required)
• \( \alpha \) decay: less exothermic, but much lower initiation energy ("self-initiating" with tunneling)

![Internuclear PE Graphs](image)

Visualization of relative distance and energy scales involved in different nuclear processes

• Uses already available information:
  Nuclear radius, calculated from nucleon number
  \( Q \)-values, calculated from mass decrements

• Emphasizes reversibility of exo/endo- thermic fission/fusion processes

• Quantifies net internuclear forces during elongation/contraction phases of fission/fusion

• Graphical comparison of initiation energies for:
  Induced versus spontaneous fission
  Stellar nucleosynthesis processes
  \( \alpha \) decay versus "symmetric" fission

• More complex effects can be introduced and incorporated (such as tunneling, smoother increase in internuclear \( PE \) due to increase in surface area during elongation)

• Conclusions
  So what can internuclear \( PE \) graphs do?

Conclusions
  Visualization of relative distance and energy scales involved in different nuclear processes

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